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MATHEMATICAL AND COMPUTER MODELING OF THE STRESS-STRAIN STATE OF THE ROCK MASS COMPOSED OF TWO ROCK TYPES NEAR THE CIRCULAR PRESSURE EXCAVATION

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Research aim *is to get the analytical solution of geomechanics topical problem of applied significance in hydrotechnical construction connected with the stress-strain state estimation of the rock mass composed of two rock types with various stress-strain behaviours in the vicinity of the circular pressure excavation located near the rectilinear interface between rocks.*
Research methodology *is based on the application of the complex variable functions theory, the properties of series with complex coefficients, and the integrals of Cauchy type.*
Research results *is the complete calculation algorithm developed, featuring iterative process computer implementation, the first convergence of which considers the well-known problem for circular opening within infinite homogenous isotropic medium under internal pressure; and the influence of the interface between rocks is considered by means of setting additional pressures which are defined more precisely at the following iterations. The criterion for the iterative process completion is satisfaction with the required accuracy of all boundary conditions assigned (during calculation, maximum inaccuracy didn't exceed* 10^{-5}*). The obtained results were used to develop computer model of the problem under consideration which has been implemented in the software package of the FEM (finite elements method) with the aim of specifying the sizes of the modeling domain and for boundary conditions formation. Finally, the comparison of analytical and numerical modeling results is introduced as applied to the specific excavation.*
Conclusion. *The analysis of the obtained results allowed to conclude that the high accuracy of calculation with the use of FEM is achieved only in case of setting sufficient research area (each size must significantly exceed five radii of excavation).*
Obtained results application area *is the development of underground structures calculation design methodology.*

Key words: *rock mass; pressure excavation; theory of elasticity, boundary conditions; computer modeling.*

Introduction. The forecast of the stress-strain state of the rock mass in the vicinity of pressure excavations is an important applied problem which is considered within the context of geomechanics as long as the obtained results create theoretical foundation for applied problems solution connected with hydrotechnical tunnels underground structures design. This accounts for a multitude of recently made analytical, numerical, and field investigations dedicated to the indicated important scientific-engineering problem [1–7]. It should be noted that the development of software technologies which significantly raised the possibilities of not only numerical modeling of various geomechanical processes within the rock mass, but also predetermined the development of analytical calculation methods which play important role in analyzing the regularities in pressure fields formation around mine workings. Thus, due to the use of modern computer hardware, there appeared the possibility to forgo complicated mathematical transformations; it has significantly simplified the process of obtaining calculation forms representing strict analytical solutions not by closed formulae but by computation algorithms which, as a rule, implement iterative processes where the functions which should be defined are represented as series. At that, the accuracy of the calculation process can always be controlled in each convergence, and the final accuracy of the calculation is checked upon completion as a result of assessing the inaccuracy in satisfying boundary conditions.

Up to the present, analytical methods of calculation in geomechanics reduced to the analysis of situations where the rock mass surrounding the excavation was modeled with homogenous medium or possessed so called technological inhomogeneity which meant circular variation of deformation and strength properties of rocks in the vicinity of the excavation as a result of technological factors influence, for example, massif weakening as a result of drilling and blasting influence, or, on the contrary, strengthening by means of injecting binder solutions into the massif [8]. Because of the lack of the corresponding analytical methods, the influence of bedded structure of the rock masses composed of various rock types was accounted only on the basis of computer modeling with the use of numerical methods, the most accepted of which was the method of finite elements (FEM) [9].

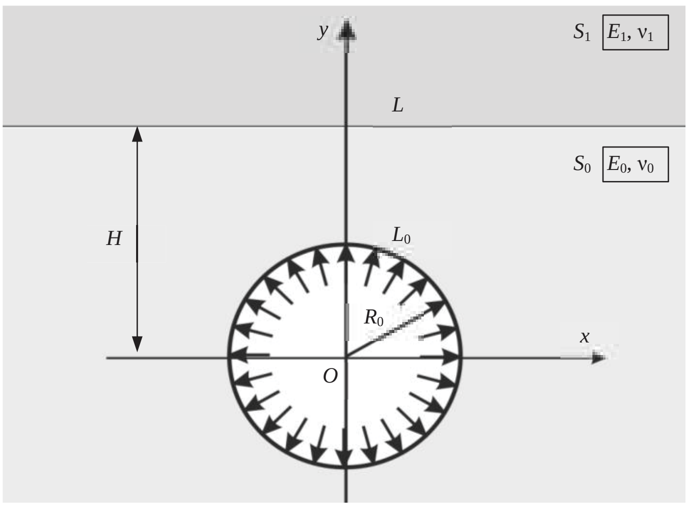

Fig. 1. Rock mass modeling
Рис. 1. Моделирование горного массива

Setting the research problem. The current research suggests new analytical solution to the problem of geomechanics on the stressed-strain state of the rock mass composed of two rock types with various stress-strain behaviours in the vicinity of circular pressure excavation located near rectilinear interface between rocks. The obtained solution allowed to fulfill the comparative analysis of mathematical and numerical modeling results and formulate specific recommendations for the development of computer model implemented in FEM software system which allowed to obtain the fullest compliance between calculation stress fields and displacements around the excavation in the particular case under consideration.

The rock mass is modeled with the area composed of two semi-finite media S_0 and S_1 modeling corresponding rock beds with rectilinear interface line L (calculation scheme of the set problem is introduced at fig. 1). Pressure excavation constructed below the interface between the rocks at the depth H is modeled with the circular opening with the radius R_0, the contour of which is subject to uniform normal pressure P.

Rock beds, areas S_j (j = 0.1), possess various stress-strain behaviours – deformation modules E_j (j = 0.1) and Poisson coefficients ν_j (j = 0.1) and are deformed collectively, i.e. at the interface line L the conditions of normal and shearing stresses and displacements vectors continuity are met.

To solve the indicated problem, the method suggested in the research [10] has been applied, which is based on the complex variables analytical functions theory, which has been modified with regard to the features of the calculation scheme under consideration.

Analytical solution of the problem. Cartesian coordinate system xOy is introduced, the origin of which is located at the center of the opening (excavation). The direction of the real axis Ox is specified parallel to the interface L. After assigning all geometrical dimensions to the excavation radius R_0, the coordinates of the points t belonging to the interface L will be defined according to the formula:

$$t = x + ih,$$

where h is the relative distance from the center of the excavation to the interface L, $h = H/R_0$.

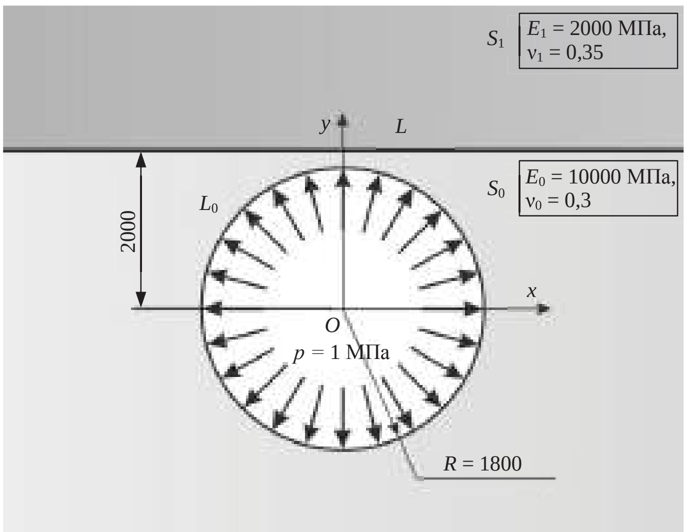

Fig. 2. Distribution of stresses in the vicinity of the pressure excavation
Рис. 2. Распределение напряжений в окрестности напорной выработки

Stress-strained state of the media S_j (j=0.1) is defined with the help of complex potentials of Kolosov and Muskhelishvili [11] $\tilde{\varphi}_j(z)$, $\tilde{\psi}_j(z)$ which are represented as

$$\tilde{\varphi}_j(z) = \varphi_{0,0}(z) + \varphi_j(z); \qquad \tilde{\psi}_j(z) = \psi_{0,0}(z) + \psi_j(z), \tag{1}$$

where $\varphi_{0,0}(z)$, $\psi_{0,0}(z)$ are the analytical functions which are regular in a complete plane S_0+S_1 beyond the opening and disappear at infinity (at that stress-strain behaviours of the examined infinite area are accepted the same as in the lower medium S_0); $\varphi_j(z)$, $\psi_j(z)$, where j = 0.1 are the functions which are regular in the corresponding half-planes S_j with the help of which the account of the presence of infinite line L is accounted – an interface between the beds with various stress-strain behaviours.

Boundary conditions at the interface L which divides the layers are written as [11]

$$\begin{gathered}\bar{\varphi}_1(t)+t\overline{\bar{\varphi}_1'(t)}+\overline{\bar{\psi}_1(t)}=\bar{\varphi}_0(t)+t\overline{\bar{\varphi}_0'(t)}+\overline{\bar{\psi}_0(t)};\\ æ_1\bar{\varphi}_1(t)-t\overline{\bar{\varphi}_1'(t)}-\overline{\bar{\psi}_1(t)}=\frac{\mu_1}{\mu_0}\left[æ_0\dot{\varphi}_0(t)-t\overline{\bar{\varphi}_0'(t)}-\overline{\bar{\psi}_0(t)}\right],\end{gathered} \tag{2}$$

where $æ_j=3-4\nu_j$, $\mu_j=E_j\left[2\left(1+\nu_j\right)\right]^{-1}$.

At the contour of the excavation L_0 the boundary condition is as follows:

$$\bar{\varphi}_1(t)+t\overline{\bar{\varphi}_1'(t)}+\overline{\bar{\psi}_1(t)}=-pt.$$

It is easy to note that if stress-strain behaviours of media S_j are set the same in the calculation scheme of the problem under consideration (fig. 1), i. e. assume $E_0=E_1$, $\nu_0=\nu_1$, then conditions (2) reduce to identities which result in equalities:

$$\varphi_j(z)=0; \qquad \psi_j(z)=0. \tag{3}$$

On the other hand, having imposed condition (3) as the first convergence of a solution we have been led to the well-known problem about the stress- strained state of the elastic plane weakened by a circular opening, the contour of which is subject to uniform internal pressure. Complex potentials $\varphi_{0,0}(z)$, $\psi_{0,0}(z)$ in this case may be represented as series:

$$\varphi_{0,0}(z)=\sum_{\nu=1}^{\infty}c_\nu^{(1)(0)}z^{-\nu}; \qquad \psi_{0,0}(z)=\sum_{\nu=0}^{\infty}c_\nu^{(2)(0)}z^{-\nu}, \tag{4}$$

where $c_\nu^{(j)(0)}$ are the sought coefficients under $j=1, 2$.

In the following, taking into account (1) and applying operations similar to the ones described in the research work, as a result of conditions (2) transformation, and through the potentials $\varphi_{0,0}(z)$, $\psi_{0,0}(z)$ it is possible to arrive to the expressions for the remaining sought functions:

$$\begin{gathered}\varphi_0(z)=-\frac{d}{s}\left[z\overline{\varphi_{0,0}'}(z-2ih)+\overline{\psi_{0,0}}(z-2ih)\right];\\ \psi_0(z)=-\frac{l}{n}\overline{\varphi_{0,0}}(z-2iH)-(z-2iH)\varphi_0'(z);\\ \varphi_1(z)=d\left[z\overline{\varphi_{0,0}'}(z-2ih)+\overline{\psi_{0,0}}(z-2ih)\right]+(s-1)\varphi_{0,0}(z);\\ \psi_1(z)=-(z-2ih)\varphi_1'(z)+l\overline{\varphi_{0,0}}(z-2ih)+(n-1)\left[(z-2iH)\varphi_{0,0}'(z)+\psi_{0,0}(z)\right],\end{gathered} \tag{5}$$

where $s=\left(1+\frac{\mu_1}{\mu_0}æ_0\right)\cdot\left(æ_1+1\right)^{-1}$; $\quad d=\left(1-\frac{\mu_1}{\mu_0}\right)\cdot\left(æ_1+1\right)^{-1}$;

$l=\left(æ_1-\frac{\mu_2}{\mu_1}æ_0\right)\cdot\left(æ_1+1\right)^{-1}$; $\quad n=\left(æ_1+\frac{\mu_1}{\mu_0}\right)\cdot\left(æ_1+1\right)^{-1}$.

Thus, as a result of expressions (4) substitution in the correlations (5) the solution of the said problem may be reduced to the search for the two groups of unknown quantities $c_{\nu}^{(k)(0)}$ $(k = 1, 2;\ \nu = 0,\ \ldots,\ \infty)$ through which all the sought functions are expressed which determine stress-strained state of the areas under consideration.

Having limited infinite series (4) up to N members we present the process of calculations in the form of the algorithm which implements a well converging iteration process the first convergence of which is build through meeting conditions (3) and solving the problem for an infinite plane with a circular opening the contour of which is subject to uniform pressure [12]. The coefficients $c_{\nu}^{(k)(0)}$ found in the first convergence are substituted in the formulae (4) and (5), and the process of calculations continues until the distinctions between the sought coefficients found in the previous and the following iterations are lower than the pre-determined small value which determines the accuracy of the calculations, for example, 10^{-6}. After the coefficients of the series (4) are found, potentials (1) are being determined, and after that it is possible to calculate stresses and displacements in the areas under consideration which model the mass composed of two rock types using the formulae of Kolosov and Muskhelishvili.

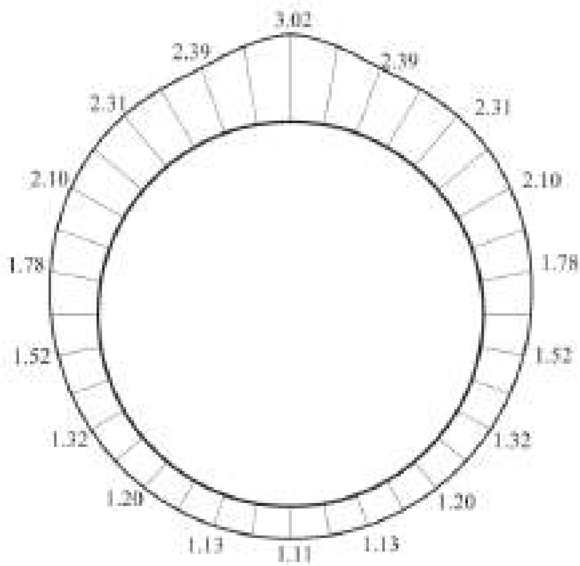

Fig. 3. Stress diagram σ_{θ}/P at the contour of the excavation
Рис. 3. Эпюра напряжений σ_{θ}/P на контуре выработки

Comparative analysis of analytical solution and numerical modeling results. The described solution is implemented in the form of a complete algorithm and computer program which allows making multivariant calculations with the purpose of analyzing stress-strained state of soil mass in the vicinity of a circular excavation.

It can be easily noted that in a particular case accepting $E_1 = 0$ and $\nu_1 = 0.5$ as initial data we come to a well-known solution of a corresponding problem for a shallow pressure excavation [12].

Further, as an example, the definition of tangential (circumferential) stresses in the vicinity of a pressure excavation (fig. 2) is considered, obtained as a result of calculating by the suggested method and computer modeling with the method of finite elements under the following data $R_0 = 1.8$ m; $H = 2$ m; $E_0 = 10\ 000$ MPa; $\nu_0 = 0.3$; $E_0 = 2000$ MPa; $\nu_0 = 0.35$.

As long as in the process of computer modeling the applied two-dimensional computational scheme is one of the simplest in geomechanics, it is considered that its implementation should be standard, and due to the load being self-balanced it is possible to forgo implicit techniques ensuring model's balance. At that it was curious whether the complete correspondence between the obtained results occurs at once, or in the particular given case there will be the need for the model's correction (for example, it will be necessary to expand the area under consideration) in order to reduce the divergence between the results of analytical and numerical calculations up to the acceptable values.

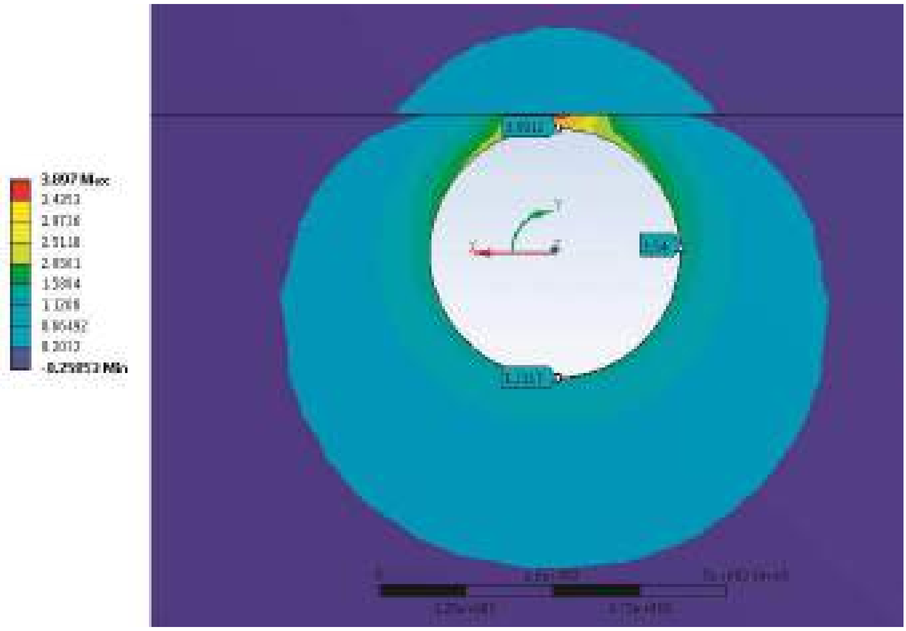

Fig. 4. Isofields of circular stresses σ_θ around the excavation
Рис. 4. Изополя окружных напряжений σ_θ вокруг выработки

Stress diagram obtained as a result of calculating by the suggested method is presented at fig. 3 in a dimensionless form (in the initial pressure value P proportions).

In the process of computer modeling the dimensions of the area under consideration had to be expanded up to 20 × 22 m. At that, satisfactory results agreement has been obtained under various boundary conditions (divergence in each considered case didn't differ by more than 5%). Isofields of normal tangential stresses in the vicinity of the excavation are presented at fig. 4. $P = 1$ MPa was accepted as a load in the process of modeling.

Conclusion. As follows from the calculation schemes introduced, divergence between mathematical and computer modeling results in the considered case doesn't exceed 8%. It should be noted that high accuracy of the calculation with the use of FEM has been achieved as a result of solving the problem of a significantly large exploration area each dimension of which significantly exceeds $5R$.

REFERENCES

1. Sammal' A. S., Antsiferov S. V., Deev P. V. *Analiticheskie metody rascheta podzemnykh sooruzhenii* [Analytical methods of calculating underground structures]. Tula, TulSU Publ., 2013. 115 p.

2. Zhaoa G. S., Zhonga G. R., Zhua F. P., Lianga H. C. Analysis of stratum grouting influence on shaft lining stress with the methods of simulation and in site measurements. Proc. Earth and Planetary Science, 2009, pp. 497–502.

3. Strokova L. A., Dutova E. M., Ermolaeva A. V., Alimova I. N., Strelnikova A. B. Karst hazard assessment in the design of the main gas pipeline (South Yakutia). IOP Conference Series: Earth and Environmental Science, 2015, vol. 27. 012032.

4. Sousa L. R., Sousa R. L., Vargas Jr. E., Velloso R., Karam K. Risk assessment on CO_2 injection processes and storage. *Chapter for Rock Mechanics and Rock Engineering*, 2017, vol. 3, chap. 12, pp. 359–397.

5. Pleshko M. S., Stradanchenko S. G., Maslennikov S. A., Pashkova O. V. Study of technical solutions to strengthen the lining of the barrel in the zone of influence of construction near-wellbore production. *ARPN Journal of Engineering and Applied Sciences*, 2015, vol. 10, no. 1, pp. 14–19.

6. Prokopov A. Iu., Sklepchuk V. L. [Technological features of sinking and lining of vertical shafts in the presence of oil and gas]. *Gornyi informatsionno-analiticheskii biulleten' (nauchno-tekhnicheskii zhurnal) – Mining Informational and Analytical Bulletin (scientific and technical journal)*, 2013, no. 11, pp. 137–145. (In Russ.)

7. Aitaliev Sh. M., Banichuk N. B., Kaiupov M. A. *Optimal'noe proektirovanie protiazhennykh podzemnykh sooruzhenii* [Extended underground structures design optimization]. Alma-Ata, 1986. 229 p.

8. Fotieva N. N., Sammal' A. S. Evaluation of opening stability with rocks technological heterogeneity being taken into account. Proc. Conf. “Geomechanics 93”, Ostrava, 1993, pp. 189–192.

9. Amusin B. Z., Fadeev A. B. *Metod konechnykh elementov pri reshenii zadach gornoi geomekhaniki* [Finite elements method at solving the problems of geomechanics]. Moscow, Nedra Publ., 1975. 144 p.

10. Sammal' A. S., Khrenov S. I. [Calculating noncircular shallow tunnels variable thickness lining]. *Gornyi informatsionno-analiticheskii biulleten' (nauchno-tekhnicheskii zhurnal) – Mining Informational and Analytical Bulletin (scientific and technical journal)*, 2004, no. 12, pp. 209–212. (In Russ.)

11. Muskhelishvili N. I. *Nekotorye osnovnye zadachi matematicheskoi teorii uprugosti* [Some basic problems of mathematical elasticity theory]. Moscow, Nauka Publ., 1969. 700 p.

12. Bulychev N. S. *Mekhanika podzemnykh sooruzhenii v primerakh i zadachakh: ucheb. posobie dlia vuzov* [School Book for the Institutions of Higher Education “Underground structures mechanics with examples and problems”]. Moscow, Nedra Publ., 1989. 270 p.

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МАТЕМАТИЧЕСКОЕ И КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ГОРНОГО МАССИВА, СЛОЖЕННОГО ДВУМЯ ТИПАМИ ПОРОД, В ОКРЕСТНОСТИ НАПОРНОЙ КРУГОВОЙ ВЫРАБОТКИ

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Целью работы *является получение аналитического решения актуальной задачи геомеханики, имеющей важное прикладное значение в гидротехническом строительстве, которая связана с оценкой напряженно-деформированного состояния горного массива, сложенного двумя типами пород с различными деформационными характеристиками, в окрестности круговой напорной выработки, расположенной вблизи прямолинейной границы раздела пород.*
Методология проведения исследования *базируется на применении теории функций комплексного переменного, свойств рядов с комплексными коэффициентами и интегралов типа Коши.*
Результатом выполненного исследования *является разработанный полный алгоритм расчета, особенностью которого является компьютерная реализация итерационного процесса, где в первом приближении рассматривается известная задача для кругового отверстия в бесконеч-ной однородной изотропной среде при действии внутреннего давления, а влияние границы раздела пород учитывается путем задания дополнительных напряжений, уточняемых на следующих итерациях. Критерием для завершения итерационного процесса является удовлетворение с заданной точностью всех поставленных граничных условий (при проведении расчетов максимальная*

погрешность не превышала 10^{-5}). Полученные результаты использованы при построении компьютерной модели рассматриваемой задачи, реализованной в программном комплексе МКЭ (метод конечных элементов) с целью уточнения размеров области моделирования и формирования граничных условий. В заключение приводится сравнение результатов аналитического и численного моделирования применительно к конкретной выработке.
Вывод. *Анализ полученных результатов позволил заключить, что высокая точность расчета с применением МКЭ достигается только при задании достаточно большой области исследования (каждый из размеров должен существенно превышать пять радиусов выработки).*
Область применения полученных результатов *– совершенствование методологии расчета и проектирования подземных сооружений.*

Ключевые слова: *горный массив; напорная выработка; теория упругости; граничные условия; компьютерное моделирование.*

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БИБЛИОГРАФИЧЕСКИЙ СПИСОК

1. Саммаль А. С., Анциферов С. В., Деев П. В. Аналитические методы расчета подземных сооружений. Тула: ТулГУ, 2013. 115 с.
2. Zhaoa G. S., Zhonga G. R., Zhua F. P., Lianga H. C. Analysis of stratum grouting influence on shaft lining stress with the methods of simulation and in site measurements. Proc. Earth and Planetary Science. 2009. P. 497–502.
3. Strokova L. A., Dutova E. M., Ermolaeva A. V., Alimova I. N., Strelnikova A. B. Karst hazard assessment in the design of the main gas pipeline (South Yakutia). IOP Conference Series: Earth and Environmental Science. 2015. Vol. 27. 012032.
4. Sousa L. R., Sousa R. L., Vargas Jr. E., Velloso R., Karam K. Risk assessment on CO_2 injection processes and storage. Chapter for Rock Mechanics and Rock Engineering. 2017. Vol. 3. Chap. 12. P. 359–397.
5. Pleshko M. S., Stradanchenko S. G., Maslennikov S. A., Pashkova O. V. Study of technical solutions to strengthen the lining of the barrel in the zone of influence of construction near-wellbore production. ARPN Journal of Engineering and Applied Sciences. 2015. Vol. 10. No. 1. P. 14–19.
6. Прокопов А. Ю., Склепчук В. Л. Технологические особенности проходки и крепления вертикальных стволов на участках нефтегазовых проявлений // ГИАБ. 2013. № 11. С. 137–145.
7. Айталиев Ш. М., Баничук Н. Б., Каюпов М. А. Оптимальное проектирование протяженных подземных сооружений. Алма-Ата, 1986. 229 с.
8. Fotieva N. N., Sammal A. S. Evaluation of opening stability with rocks technological heterogeneity being taken into account. Proc. Conf. “Geomechanics 93”. Ostrava, 1993. P. 189–192.
9. Амусин Б. З., Фадеев А. Б. Метод конечных элементов при решении задач горной геомеханики. М.: Недра, 1975. 144 с.
10. Саммаль А. С., Хренов С. И. Расчет обделок переменной толщины некруговых тоннелей мелкого заложения // ГИАБ. 2004. № 12. С. 209–212.
11. Мусхелишвили Н. И. Некоторые основные задачи математической теории упругости. М.: Наука, 1969. 700 с.
12. Булычев Н. С. Механика подземных сооружений в примерах и задачах: учеб. пособие для вузов. М.: Недра, 1989. 270 с.

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